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3 (Sem 2) MAT M1

2015

MATHEMATICS

(Major)

Paper : 2.1

(Co-ordinate Geometry)

Full Marks – 80

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. (a) Transform the equation  $y^2 - 4x + 4y + 8 = 0$  to parallel axes through the point  $(1, -2)$ . 1
- (b) What is the angle between the lines represented by the equation  $x^2 - y^2 = 0$  ? 1
- (c) What is the equation of the tangent to the parabola  $y^2 = 4ax$  at the point  $(x_1, y_1)$  ? 1
- (d) What is the eccentricity of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  ( $b > a$ ) ? 1

[Turn over

- (e) Write down the asymptotes of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad 1$$

- (f) What are the direction ratios of the normal to the plane

$$x + y + z = 1 ?$$

- (g) Define skew lines. 1

- (h) Write down the centre and radius of the sphere given by the equation 1

$$x^2 + y^2 + z^2 - 2x - 4y + 6z = 22$$

- (i) What is the general equation of a second degree cone passing through the coordinate axes ? 1

- (j) Define enveloping cylinder. 1

2. (a) Find the transformed equation of the line  $y = x$  when the axes are rotated through an angle  $45^\circ$ . 2

- (b) Find the value of  $K$  so that the following equation may represent pair of straight lines -

$$Kxy - 8x + 9y - 12 = 0 \quad 2$$

- (c) Find the equation of the cone whose vertex is at the origin and whose guiding curve is given by

$$x = a, \quad y^2 + z^2 = b^2 \quad 2$$

- (d) If  $(at_1^2, 2at_1)$  and  $(at_2^2, 2at_2)$  are the extremities of any focal chord of the parabola  $y^2 = 4ax$ , prove that  $t_1 t_2 = -1$  2

- (e) Find the equation of the sphere through the circle  $x^2 + y^2 + z^2 = 9$ ,  $2x + 3y + 4z = 5$  and the origin. 2

3. (a) Transform the equation

$3x^2 + 8xy + 3y^2 - 2x + 2y - 2 = a^2$  referred to a new set of axes through  $(-1, 1)$  rotated through an angle  $\frac{\pi}{4}$ . 5

- (b) Show that the equation

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$  represents a pair of parallel straight lines if

$$\frac{a}{h} = \frac{h}{b} = \frac{g}{f} \quad 5$$

Or

Show that the straight lines joining the origin to the other two points of intersection of the curves whose equations are

$$ax^2 + 2hxy + by^2 + 2gx = 0 \quad \text{and}$$

$$a'x^2 + 2h'xy + b'y^2 + 2g'x = 0$$

will be at right angles if  $g(a'+b') = g'(a+b)$

4. (a) Prove that the line  $lx + my = n$  is a normal to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  if

$$\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2 - b^2)^2}{n^2}$$

- (b) A sphere of constant radius  $r$  passes through the origin  $O$  and cut the axes at  $A$ ,  $B$  and  $C$ . Prove that the locus of the foot of the perpendicular from  $O$  to the plane  $ABC$

$$(x^2 + y^2 + z^2)^2 \left( \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} \right) = 4r^2$$

Or

Show that the shortest distance between any two opposite edges of the tetrahedron formed by the planes

$$y + z = 0, \quad z + x = 0, \quad x + y = 0, \quad x + y + z = a$$

is  $\frac{2a}{\sqrt{6}}$  and that the three lines of shortest distance intersect at the point  $(-a, -a, -a)$

5. Answer any four parts :

5×4=20

- (a) Show that the equation of the tangent to the conic

$$\frac{l}{r} = 1 + e \cos \theta$$

at the point whose vectorial angle is  $\alpha$  is

$$\frac{l}{r} = e \cos \theta + \cos(\theta - \alpha)$$

- (b) Reduce the equation

$$7x^2 - 2xy + 7y^2 - 16x + 16y - 8 = 0$$

to the standard form.

- (c) Find the lengths of the semi-axes of the conic

$$ax^2 + 2hxy + ay^2 = d$$

- (d) Prove that the equation of the polar of the origin with respect to the conic

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

is  $gx + fy + c = 0$

- (e) Prove that the middle points of chords of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

parallel to the diameter  $y = mx$  lie on the diameter  $a^2my = b^2x$ .

- (f) Find the asymptotes of the hyperbola  
 $2x^2 + 5xy + 2y^2 + 4x + 5y = 0$

6. Answer any *four* parts :

5×4

- (a) Find the condition that the two lines with equations are

$$\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1}$$

and  $\frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2}$

may intersect and also find the equation of the plane in which they lie.

- (b) Prove that the plane  $ax + by + cz = 0$  cuts the cone  $yz + zx + xy = 0$  in perpendicular

if  $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$

- (c) Find the equation of the cylinder whose generators are parallel to the line

$$2x = y = 3z$$

and which passes through the circle

$$y = 0, \quad x^2 + z^2 = 8$$

(d) Find the equation of the tangent planes to  $2x^2 - 6y^2 + 3z^2 = 5$  which pass through the line  $x + 9y - 3z = 0$ ,  $3x - 3y + 6z - 5 = 0$

(e) Find the condition that the plane

$$lx + my + nz = p$$

may be a tangent plane to the conicoid

$$ax^2 + by^2 + cz^2 = 1$$

(f) Find the condition that the line

$$\frac{x-2}{l} = \frac{y-1}{m} = \frac{z-3}{n}$$

may touch the ellipsoid

$$3x^2 + 8y^2 + z^2 = c^2.$$