2019

MATHEMATICS

(Major)

Paper : 6.5

(Graph and Combinatories)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : 1×7=7

(a) Write sum rule principle of combinatories.

(b) A farmer buys 3 cows, 2 pigs and 4 hens from a man who has 6 cows, 5 pigs and 8 hens. How many choices does the farmer have?

(c) Suppose 6 people randomly arrive at a darkroom to retrieve their hats. Find the number of ways such that no person picks his own hat.

(d) A tree

(i) is always a disconnected graph

(ii) is always a connected graph

(iii) may be connected or disconnected

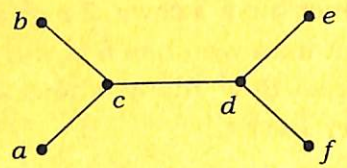
(iv) None of the above

(Choose the correct answer)

(e) Define simple graph.

(f) How many degrees of each vertex are there in a circuit?

(g) Find the eccentricity of the vertex a of the following graph :



2. Answer the following questions : 2×4=8

(a) There are 9 students in a class. Find the number of ways that the 9 students can take 3 different tests, if 3 students are to take each test.

(b) Represent the graph $G(V, E)$, where the vertex set V and edge set E are as follows :

$V = \{1, 2, 3, 4\}$, $E = \{(x, y) : |x - y| \leq 1,\ x, y \in V\}$

(c) Let v be a point of a connected graph G. If there exist points u and w distinct from v such that v is in every u–w path, then show that v is a cut point of G.

(d) Prove that every non-trivial connected graph has at least two points which are not cut points.

3. Answer the following questions : 5×3=15

(a) Show that $C(2n, 2) = 2C(n, 2) + n^2$.

(b) Prove that a simple graph with at least two vertices has at least two vertices of same degree.

Or

Prove that the sum of the degree of all vertices of a graph is an even integer.

(c) Let G be a simple graph with at most $2n$ vertices. If the degree of each vertex is at least n, then show that the graph is connected.

Or

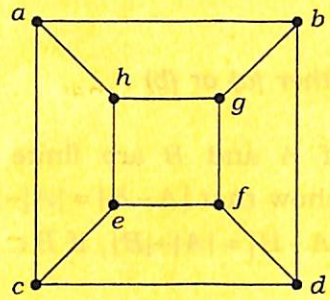
Prove that every tree has a centre consisting of either one point or two adjacent points.

4. Answer *either (a) or (b)* :

(a) Prove that a graph with at least $2n$ points is n-connected if and only if for any two disjoint sets V_1 and V_2 of n points each, there exists n disjoint path joining these two sets of points. 10

(b) (i) Show that the vertex connectivity of any graph G is always less than or equal to edge connectivity of G. 6

(ii) Find the edge connectivity and vertex connectivity of the graph G represented by



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5. Answer *either (a) or (b)* :

(a) If G is a simple graph with number of vertices $n(\geq 3)$ and if $\deg(v) + \deg(w) \geq n$ for any pair of non-adjacent vertices v and w, then show that G is Hamiltonian. 10

(b) (i) If G is a simple graph with n vertices and e edges with $n \geq 3$ and $e \geq \frac{1}{2}(n^2 - 3n + 6)$, then show that G is Hamiltonian. 5

(ii) If G is a Hamiltonian graph, then show that any non-empty proper subset S of V, $w(G - S) \leq |S|$. 5

6. Answer *either (a) or (b)* :

(a) (i) If A and B are finite sets, then show that $|A - B| = |A| - |A \cap B|$ and $|A - B| = |A| - |B|$, if $B \subset A$. 3+2=5

(ii) How many solutions are there to $x_1 + x_2 + x_3 + x_4 + x_5 = 16$, where each $x_i \geq 27$? 5

(b) (i) There are five different Hindi books, six different English books and eight different Sanskrit books. How many ways are there to pick two books not both in the same language? 4

(ii) How many solutions does $x_1 + x_2 + x_3 = 13$ have where x_1, x_2, x_3 are non-negative with $x_1 \leq 4$, $x_2 \leq 5$ and $x_3 \leq 6$? 6

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