

2014

MATHEMATICS

( Major )

Paper : 6.2

( Numerical Analysis )

Full Marks : 60

Time : 3 hours

*The figures in the margin indicate full marks  
for the questions*

1. Answer the following questions : 1×7=7
- (a) What is the normalized floating point representation of real numbers?
  - (b) Define the term relative error.
  - (c) If we take  $\pi = 3.14$  instead of  $3.14159$ , find the absolute error.
  - (d) Establish the relation,  $E \equiv 1 + \Delta$ .
  - (e) Write down the value of  $\Delta^{n+1}x^n$ .
  - (f) When are Newton's interpolation formulae used?
  - (g) What is the basic principle of numerical differentiation?

2. Answer the following questions : 2×4=8

(a) Using normalized floating point representation, add  $.4546E5$  and  $.5433E7$ .

(b) Show that

$$\Delta\{\log f(x)\} = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$$

(c) Establish the result

$$y' = \frac{dy}{dx} = \frac{1}{h}\left[\Delta y - \frac{1}{2}\Delta^2 y + \frac{1}{3}\Delta^3 y - \frac{1}{4}\Delta^4 y + \dots\right]$$

where the symbols have their usual meanings.

(d) Write the conditions under which  
(i) trapezoidal rule and (ii) Simpson's  $\frac{1}{3}$ rd rule are valid.

3. Answer the following questions : 5×3=15

(a) Explain the terms truncation error and round-off error with suitable examples.

(b) Using the method of separation of symbols, prove that

$$u_1 x + u_2 x^2 + u_3 x^3 + \dots = \frac{x}{1-x} u_1 +$$

$$\frac{x^2}{(1-x)^2} \Delta u_1 + \frac{x^3}{(1-x)^3} \Delta^2 u_1 + \dots$$

( 3 )

Or

Obtain the estimate of the missing figures in the following table :

$x$	:	1	2	3	4	5	6	7	8
$f(x)$	:	2	4	8	—	32	—	128	256

(c) Find the first derivative of the function tabulated below at the point  $x = 3.0$  :

$x$	:	3.0	3.2	3.4	3.6	3.8	4.0
$f(x)$	:	-14.000	-10.032	-5.296	0.256	6.672	14.000

Or

Evaluate  $\int_0^6 \frac{dx}{1+x^2}$  by using (i) Simpson's  $\frac{1}{3}$ rd rule and (ii) Simpson's  $\frac{3}{8}$ th rule.

4. Answer either (a) or (b) :

(a) (i) Derive the Newton's forward interpolation formula and mention when the formula gives best approximation.

4+1=5

- (ii) In an examination, the number of candidates who obtained marks between certain limits were as follows :

Marks obtained	Number of candidates
0-19	41
20-39	62
40-59	65
60-79	50
80-99	17

Estimate the number of candidates who obtained less than 70 marks. 5

- (b) (i) State and derive Stirling's central difference formula. Hence or otherwise establish Bessel's formula. 3+3=6

- (ii) Apply Bessel's formula to obtain  $y_{25}$ , given  $y_{20} = 2854$ ,  $y_{24} = 3162$ ,  $y_{28} = 3544$ ,  $y_{32} = 3992$ . 4

5. Answer either (a) or (b) :

- (a) Explain briefly the idea of numerical integration. Establish the general quadrature formula and deduce trapezoidal rule from it. 2+5+3=10

- (b) (i) The velocity  $v$ (km / min) of a vehicle which starts from rest, is given at fixed intervals of time  $t$  (min) as follows :

$t$	:	2	4	6	8	10	12	14	16	18	20
$v$	:	10	18	25	29	32	20	11	5	2	0

Estimate approximately the distance covered in 20 minutes, using Simpson's  $\frac{1}{3}$ rd rule. 5

- (ii) Compute the integral  $\int_5^{12} \frac{1}{x} dx$  by applying Gauss's quadrature formula. 5

6. Answer either (a) or (b) :

- (a) (i) State the Newton-Raphson formula and give a geometrical interpretation of it. 5
- (ii) Find a root of the equation  $x^3 - 4x - 9 = 0$ , using the bisection method correct to four decimal places. 5
- (b) (i) State the condition of convergence of Newton-Raphson method. 1

- (ii) Derive  $x_{n+1} = \frac{1}{2} \left( x_n + \frac{a}{x_n} \right)$  for determining the square root of  $a > 0$ , using Newton-Raphson formula. 2
- (iii) Show that the equation  $x^2 + \log x = 0$  has exactly one root and the root lies in the interval  $[\frac{1}{3}, 1]$ . 2
- (iv) Find a real root of the equation  $x^3 - 2x - 5 = 0$  by the method of false position correct to three decimal places. 5

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