**2016**

MATHEMATICS

( Major )

Paper : 6.1

**( Hydrostatics )**

*Full Marks* : 60

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

**1.** Answer the following questions : 1×7=7

(a) Define surface of equal pressure.

(b) If the equilibrium is stable, what is the position of metacenter with respect to the centre of gravity of a body?

(c) What happens when there is an increase of pressure at any point of a liquid at rest under given external forces?

(d) State the principle of Archimedes.

(e) Define specific heat of a body.

(f) What is an adiabatic change?

(g) State Charles' law.

**2.** Answer the following questions : $2\times4=8$

(a) Show that the surface of equal pressure is intersected orthogonally by the lines of force.

(b) Show that for a fluid under conservative forces, the surface of equal pressure is also the surface of equipotential as well as equidensity.

(c) Prove that the pressure at any point varies as the depth of the point from the surface when there is no atmospheric pressure.

(d) Define surface of buoyancy and surface of floatation.

**3.** Answer any *three* parts : $5\times3=15$

(a) If a fluid is at rest under the action of the forces $X, Y, Z$ per unit mass, find the differential equations of the curves of equal pressure and density.

(b) Prove that the position of the centre of pressure relative to the area remains unaltered by rotation about its line of intersection with the effective surface.

*(c)* Masses $m_1$ and $m_2$ of two gases in which the ratio of the pressure to the density are respectively $k_1$ and $k_2$, are mixed at the same temperature. Prove that the ratio of the pressure to the density in the compound is

$$\frac{m_1k_1 + m_2k_2}{m_1 + m_2}$$

*(d)* A solid body consists of a right cone joined to a hemisphere on the same base and floats with the spherical portion partly immersed; prove that the greatest height of the cone consistent with stability is $\sqrt{3}$ times the radius of the base.

*(e)* For an adiabatic expansion, prove that $PV^{\gamma}$ = constant, where $P$ is the pressure, $V$ is the volume and $\gamma$ is the ratio of specific heat at constant pressure and specific heat at constant volume for the gas concerned.

**4.** Answer *(a)* or *(b)* :

*(a)* *(i)* Determine the necessary condition that must be satisfied by a given distribution of forces $X$, $Y$ and $Z$ acting on a mass of fluid so that the fluid may maintain equilibrium. 5

*(ii)* Show that the forces represented by

$$X = \mu(y^2 + yz + z^2)$$
$$Y = \mu(z^2 + zx + x^2)$$
$$Z = \mu(x^2 + xy + y^2)$$

will keep a mass of liquid at rest if the density at any point is inversely proportional to the square of the distance of the point from the plane $x + y + z = 0$. 5

*(b)* *(i)* A mass of homogeneous liquid, contained in a vessel revolves uniformly about a vertical axis. Determine the pressure at any point and the surface of equal pressure. 5

*(ii)* A fluid rests in equilibrium in a field of force

$$X = y^2 + z^2 - xy - xz$$
$$Y = z^2 + x^2 - zy - xy$$
$$Z = x^2 + y^2 - xz - yz$$

Show that the curves of equal pressure and density are a set of circles. 5

**5.** Answer *(a)* or *(b)* :

*(a)* *(i)* If an area is bounded by two concentric semi-circles with their common bounding diameter in the free surface, prove that the depth of the centre of pressure is

$$\frac{3\pi(a+b)(a^2+b^2)}{16(a^2+ab+b^2)}$$

where $a$ and $b$ are the radii of the outer and inner circles respectively. 5

*(ii)* A quadrant of a circle is just immersed vertically in a heavy homogeneous liquid with one edge in the surface. Determine the position of the centre of pressure. 5

*(b)* *(i)* A vessel in the form of an elliptic paraboloid, whose axis is vertical and equation

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z}{h}$$

is divided into four equal compartments by its principal planes. Into one of these, water is poured to the depth $h$; prove that, if the resultant pressure on the curved portion be reduced to two forces, one vertical and the other horizontal, the line of action of the latter will pass through the point

$$\left(\frac{5}{16}a, \frac{5}{16}b, \frac{3}{7}h\right)$$ 5

*(ii)* A semi-ellipse bounded by its minor axis is just immersed in a liquid, the density of which varies as the depth; if the minor axis be in the surface, find the eccentricity in order that the focus may be the centre of pressure. 5

**6.** Answer *(a)* or *(b)* :

*(a)* *(i)* A hollow hemispherical shell has a heavy particle fixed to its rim, and floats in water with the particle just above the surface and with the plane of the rim inclined at an angle of 45° to the surface. Show that the weight of the hemisphere to the weight of the water which it would contain is $4\sqrt{2}-5:6\sqrt{2}$. 5

*(ii)* If a solid paraboloid, bounded by a plane perpendicular to its axis, floats with its axis vertical and vertex immersed, then show that the height of the metacenter above the centre of gravity of the displaced liquid is equal to half the latus rectum. 5

*(b)* *(i)* When the temperature is supposed to be uniform, show that as the altitude increases in arithmetical progression, pressure decreases in geometrical progression. 5

*(ii)* A straight tube, closed at one end and open at the other, revolves with a constant angular velocity about an axis meeting the tube at right angles; neglecting the action of gravity, find the density of the air within the tube at any point. 5

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