2016

PHYSICS

(Major)

Paper : 4.1

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods—IV)

1. Answer any *four* of the following questions : 1×4=4

(a) Find the value of n if

$$\int_{-1}^{+1} P_n(x)\,dx = 2$$

(b) Write the general solution of the equation

$$y'' + 2y' + 5y = 0$$

(c) Define Gaussian distribution.

(d) What is singular point in a second-order linear differential equation?

(e) If $H_n(x)$ is the Hermite polynomial, then what is the value of $H_0(x)$?

(f) What do you mean by standard deviation?

2. Answer any *three* of the following questions : $2 \times 3 = 6$

(a) Check whether Frobenius method can be applied or not to the following equation :

$$\frac{d^2y}{dx^2} - \frac{5y}{x^3} = 0$$

(b) Find the value of $P_{2n+1}(0)$.

(c) When one card is drawn from each of two decks, find the probability that at least one of them is an ace.

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(d) Find the degree and order of the following equation :

$$\frac{d^2y}{dx^2}+\frac{2}{x}\frac{dy}{dx}+\frac{9^2}{x^4}=0$$

(e) Prove that $H_1(x)=2x$.

3. Answer any *two* of the following questions : 5×2=10

(a) Use Frobenius method to find the series solution of the equation

$$\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2+2)y=0$$

(b) Show that

$$\int_{-1}^{+1}P_m(x)\,P_n(x)\,dx=0$$

where $m\neq n$.

(c) Write the generating function for Hermite polynomial $H_n(x)$ and hence show that

$$H_n(x)=(-1)^n e^{x^2}\frac{d^n}{dx^n}e^{-x^2}$$

(d) A manufacturer produces air mail envelope whose weight is normally distributed with $\mu = 1{\cdot}95$ g and standard deviation $\sigma = 0{\cdot}05$ g. The envelopes are sold in lots of 1000. How many envelopes in a lot will be heavier than 2g ? Use the fact

$$\frac{1}{\sqrt{2n}}\int_0^1 e^{-x^2/a}\,dx = 0{\cdot}3413$$

(e) Show that

$$\int_{-\infty}^{+\infty} e^{-x^2} H_m(x)\, H_n(x)\,dx = 0$$

if $m \neq n$.

4. Answer any *two* of the following questions : 10×2=20

(a) (i) Show that

$$(1-2xh+h^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x)\,h^n$$

where $P_n(x)$ is the Legendre polynomial. 5

(ii) Evaluate explicitly the Legendre Polynomials $P_2(x)$ and $P_3(x)$. 2½+2½=5

(*Continued*)

(b) Prove the following recurrence relations :

(i) $2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$ 3

(ii) $H_n'(x) = 2xH_n(x) - H_{n+1}(x)$ 3

(iii) $2nH_{n-1}(x) = H_n'(x)$ 4

(c) (i) A dice is thrown 8 times. Find the probability that '5' will show exactly twice, at least seven times and at least once. 3+3+2=8

(ii) Define total probability. 2

(d) Obtain the power series solution of the Legendre equation

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$$ 10

GROUP—B

(Introduction to Computer and Computer Programming)

5. Answer any *two* of the following : 1×2=2

(a) How will you test the efficiency of an algorithm?

(b) Name the two units of the central processing unit.

(c) What is a variable?

6. Answer any *two* of the following : 2×2=4

(a) What are the fundamental data types in C^{++}/C/FORTRAN?

(b) What is the syntax to find the number of characters in a string in C^{++}/C/FORTRAN?

(c) What are the advantages of breaking a program into subroutine?

7. Answer any *one* of the following : 4

(a) Write a note on high-level language. What are its advantages?

(b) What are the basic major computer operations or functions?

8. Answer any *one* of the following :

(a) Write the algorithm and draw the flow chart to find a prime number from a set of numbers. 5+5=10

(b) Define transfer statement. What are the different types of transfer statements? Write the syntax of all the transfer statements in C^{++}/C/FORTRAN. 1+3+6=10

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