2013

PHYSICS

(Major)

Paper : 2.1

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods II)

(*Marks* : 35)

1. Answer the following questions : 1×4=4

(a) If $\vec{F}$ is a conservative force field, find the value of $\oint_C \vec{F} \cdot d\vec{l}$ along a closed curve C.

(b) Write the mathematical statement for the flux of a vector $\vec{A}$ over a surface S.

(c) Write the expression for a vector $\vec{A}$ having covariant components c_1, c_2, c_3 in orthogonal curvilinear coordinate system (u_1, u_2, u_3).

(d) If

$$\int_{-\infty}^{+\infty} f(x)\,\delta(x)\,dx = f(0)$$

find the value of $\int_{-\infty}^{+\infty} \delta(x)\,dx$.

2. Answer the following questions : 2×3=6

(a) The equation of motion of a particle of mass m is

$$m\frac{d^2\vec{r}}{dt^2} = f(r)\,\hat{r}$$

where $\vec{r}$ is the position vector of the particle with respect to the origin and $\hat{r}$ is its unit vector. Then show that

$$\vec{r} \times \frac{d\vec{r}}{dt} = \vec{C}$$

is a constant vector.

(b) Evaluate

$$\int_2^3 \vec{A} \cdot \frac{d\vec{A}}{dt}\,dt$$

if $\vec{A}(2) = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{A}(3) = 4\hat{i} - 2\hat{j} + 3\hat{k}$.

(c) If

$$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx \quad (n > 0)$$

find $\Gamma(1)$.

3. Show that in orthogonal curvilinear coordinates

$$\vec{\nabla}\cdot(A_1\hat{e}_1)=\frac{1}{h_1h_2h_3}\frac{\partial}{\partial u_1}(A_1h_2h_3)$$

where the symbols carry usual meanings. 5

Or

Prove that $\oint d\vec{r}\times\vec{B}=\iint_S(\hat{n}\times\vec{\nabla})\times\vec{B}\,dS$ where $\hat{n}$ is the unit vector normal to the elemental area dS.

4. Answer *either (a) or [(b) and (c)]* :

Either

(a) State and prove Gauss' divergence theorem. 2+8=10

Or

(b) Express $\nabla^2\psi$ in orthogonal curvilinear coordinate system. 5

(c) If u_1, u_2 and u_3 are generalized orthogonal coordinates, then show that

$$\left(\frac{\partial\vec{r}}{\partial u_1},\frac{\partial\vec{r}}{\partial u_2},\frac{\partial\vec{r}}{\partial u_3}\right)\text{ and }(\vec{\nabla}u_1,\vec{\nabla}u_2,\vec{\nabla}u_3)$$

are reciprocal system of vectors. 5

5. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] :

Either

(a) Verify Stokes' theorem for

$$\vec{A} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$$

taking S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. 7

(b) Show that

$$\delta(ax) = \frac{1}{a}\delta(x), \qquad a > 0$$ 3

Or

(c) Calculate the scale factors h_1, h_2, h_3 in spherical polar coordinates. 4

(d) Show that

$$\Gamma\left(\frac{1}{2}\right) = \pi^{\frac{1}{2}}$$

Hence calculate $\Gamma\left(\frac{5}{2}\right)$. 5+1=6

GROUP—B

(Properties of Matter)

(*Marks* : 25)

6. Answer the following questions : 1×3=3

(a) What is the physical interpretation of negative value of Poisson's ratio?

(b) Why is the raindrop spherical?

(c) When a liquid in a tube is stirred and left itself, the motion subsides after sometime. For what phenomenon does it happen?

7. A steel rod of length 50 cm, width 2 cm and thickness 1 cm is bent in the form of an arc having radius 2·0 m. If Young's modulus of the material of the rod is 2×10^{11} N/m^2, calculate the bending moment. 2

8. Answer any *two* of the following questions : 5×2=10

(a) Show that a hollow cylinder is stronger than a solid one of the same mass, length and material.

(b) Find an expression for the volume of liquid flow per unit time through a combination of two capillary tubes of lengths and radii (l_1, r_1) and (l_2, r_2) joined smoothly in series.

(c) Derive an expression for excess pressure inside a curved surface of a liquid.

9. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] of the following questions :

Either

(a) Derive the relation $Y = 2n(1+\sigma)$, where Y, n and σ are Young's modulus, rigidity modulus and Poisson's ratio of a material respectively. 7

(b) A U-tube is supported with its limbs vertical and is partly filled with water. If the internal diameters of the limbs are 1 cm and 1 mm, what will be the difference in heights at which the water stands in two limbs? [Take surface tension of water = 70 dynes/cm, $g = 980$ cm/s^2 and density of water $\rho = 1$ gm/cc.] 3

Or

(c) Show that a cantilever clamped at one end and loaded at other end executes a simple harmonic oscillation, when it is depressed slightly and then released. 7

(d) In an experiment with Poiseuille's apparatus, the following readings were recorded :

Volume of alcohol flowing per minute $= 10^{-5}$ m^3

Density of alcohol $= 8 \times 10^2$ kg/m^3

Length of tube = 0·5 m

Radius of tube = 0·05 cm

Height of alcohol head = 0·6 m

Determine the coefficient of viscosity of the alcohol. 3

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