2016

PHYSICS

(Major)

Paper : 2.1

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods–II)

(*Marks* : 35)

1. Answer the following questions : 1×4=4

   (a) Find the value of $\vec{r}$ satisfying the equation $\left(\dfrac{d^2\vec{r}}{dt^2}\right) = a$.

   (b) What are the coordinate surfaces in cylindrical coordinates?

   (c) Find the value of $\Gamma(0)$.

   (d) Evaluate : $\int_2^6 (3x^2 - 2x - 1)\,\delta(x-3)\,dx$

2. Answer the following questions : 2×3=6

(a) Find the total work done in moving a particle in a force field given by $\vec{F} = 3xy\hat{i} - 5z\hat{j} + 10x\hat{k}$ from $t = 1$ to $t = 2$.

(b) Define Gamma function and Dirac delta function.

(c) Show that $\iint_S \vec{A} \cdot \hat{n}\, dS$, over any closed surface S is equal to $\iint_R \vec{A} \cdot \hat{n} \frac{dx\,dy}{|\hat{n} \cdot \hat{k}|}$, where R is the projection of S on xy-plane.

3. Answer any *two* of the following questions : 3×2=6

(a) If $\vec{F} = (2x^2 - 3z)\hat{i} - 2xy\hat{j} - 4x\hat{k}$, then evaluate $\iiint_V \vec{\nabla} \cdot \vec{F}\, dV$, where V is the region bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $2x + 2y + z = 4$.

(b) Show that in orthogonal coordinates

$$\vec{\nabla} \times (A_1 \hat{e}_1) = \frac{\hat{e}_2}{h_3 h_1} \frac{\partial}{\partial u_3}(A_1 h_1) - \frac{\hat{e}_3}{h_1 h_2} \frac{\partial}{\partial u_2}(A_1 h_1)$$

where symbols have their usual meanings.

(c) Show that $\Gamma(n) = (n-1)\Gamma(n-1)$ for all values of n and $\Gamma(n) = (n-1)!$, when n is a positive integer.

4. Answer any *one* of the following : 4

(a) Verify divergence theorem for the vectors $\vec{V} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$, taken over the cube $0 < x, y, z < 1$.

(b) Prove that a spherical coordinate system is orthogonal.

5. Answer any *three* questions from the following : 5×3=15

(a) Show that—

(i) $\delta(-x) = \delta(x)$

(ii) $x\delta'(x) = -\delta(x)$ 2+3=5

(b) Show that $\iint_S (\vec{\nabla} \times \vec{A}) \cdot \hat{n} dS = \oint_C \vec{A} \cdot d\vec{r}$

(c) Express $\nabla^2\psi$ in orthogonal curvilinear coordinates.

(d) If S be a closed surface and $\vec{r}$ denotes the position vector of any point (x, y, z) measured from an origin O, prove that $\iint_S \frac{\hat{n} \cdot \vec{r}}{r^3} dS$ is equal to 4π if O lies inside S.

(e) Find the circulation of F round the curve C, where

$$\vec{F} = e^x \sin y\hat{i} + e^x \cos y\hat{j}$$

and C is the rectangle whose vertices are $(0, 0)$, $(1, 0)$, $(1, \frac{\pi}{2})$ and $(0, \frac{\pi}{2})$.

GROUP—B

(Properties of Matter)

(*Marks* : 25)

6. Answer the following questions : 1×5=5

(a) Why is a hollow shaft of the same material, mass and length very much stronger than that of a solid shaft?

(b) What is the velocity profile of a advancing liquid through a horizontal capillary tube?

(c) When is the surface energy of a liquid surface equal to the surface tension?

(d) Write the expression for excess pressure of cylindrical bubble in a liquid.

(e) Write the expression for time period of a torsional pendulum.

7. Answer *either (a)* and *(b) or (c)* and *(d)* : 10

Either

(a) Find an expression for energy per unit volume of a stretching wire. Show that a shear is equivalent to a compression and an extension. 3+3=6

(b) A bronze bar, 1·7 m long and 50 mm in diameter is subjected to a tensile stress of 70 meganewton/m^2. Calculate the extension produced in the bar and the work done during the process. The value of Young's modulus for the material of the bar may be taken to be 85×10^9 N/m^2. 4

Or

(c) Calculate the depression at the free end of a thin light beam, clamped horizontally at one end and loaded at the other. 5

(d) For the same cross-sectional area, show that a beam of square section is stiffer than that of a circular section of same material. Show also for a given load the depressions are in the ratio $3 : \pi$. 5

8. Answer any *two* questions of the following : 5×2=10

(a) Show that shearing stress at a point in a twisted cylinder or a wire depends on the distance of the point from the axis and not its vertical distance from either end of it. 5

(b) The rate of flow of liquid through a capillary is given $Q = \frac{\pi P r^4}{8 \eta l}$ with usual notations. Deduce this relation stating clearly the conditions under which it holds. Why does the formula fail in the case of tube of wide bore? 4+1=5

(c) *(i)* For a homogeneous isotropic substance, show that $\frac{Y}{\eta} = 2(\sigma + 1)$, where symbols have their usual meanings.

(ii) A gold wire 0·32 mm in diameter elongated by 1 mm, when stretched by a force of 330 gm-wt and twists through 1 radian, when equal an opposite torque of 145 dyne-cm are applied at its end. Find the value of Poisson's ratio for gold. 3+2=5

(d) *(i)* Show that excess pressure acting on a curved surface of a curved membrane is given by

$$P = 2T\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

where r_1 and r_2 are the radii of curvature and T the surface tension of the membrane.

(ii) Two equal spherical soap bubbles coalesce to form one spherical soap bubble without any leakage of air. If V is the consequence change in volume of the contained air and S, the change in the total surface area, show that $3PV + 4ST = 0$, where T is the surface tension of the soap bubble and P, the atmospheric pressure. 3+2=5

★★★