2013

PHYSICS

(Major)

Paper : 1.1

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods)

(*Marks* : 20)

1. (a) What do we mean by saying that a vector $\vec{Q}$ is constant? 2

(b) Show that $\vec{r}\,(\vec{\nabla}\cdot\vec{r}) \neq (\vec{r}\cdot\vec{\nabla})\,\vec{r}$. 2

(c) Suppose on classical consideration a fundamental particle spins about its own axis of rotation and another does not spin. Can you differentiate the two particles in terms of scalar and vector particle? Give answer with explanation. 4

(d) Find the projection of $\vec{A} = \hat{i} + 2\hat{j} + \hat{k}$ on $\vec{B} = \hat{j} + 2\hat{k}$. 2

2. (a) The motion of a particle is described by giving x, y, z as functions of t, where the symbols carry usual meanings. Let $x = c\cos\omega t$, $y = c\sin\omega t$, $z = 0$, where c and ω are constants. Find $\vec{r}$, $\vec{v}$ and $\vec{a}$, and comment on the motion, the direction of velocity and the acceleration respectively. 4

(b) How are the components of a vector given in terms of the magnitude and the direction cosines of the vector? How do you consider the general space as scalar field or vector field? Explain grad V, where $V(x, y, z)$ is scalar field. Find the components of grad V on the axes of coordinates. 1+1+1+3=6

OR

3. (a) $\vec{A} \times \vec{B} = -m\vec{B} \times \vec{A}$, where m is a constant. Give one physical concept that can be assigned to this mathematical statement. 1

(b) Show that the gradient of any scalar field $\phi(\vec{r})$ is irrotational and that the curl of any vector field $V(\vec{r})$ is solenoidal. 3

(c) Show that $1 = \cdot\hat{e}_x\hat{e}_x + \cdot\hat{e}_y\hat{e}_y + \cdot\hat{e}_z\hat{e}_z$, where $\hat{e}_x$, $\hat{e}_y$ and $\hat{e}_z$ are the unit vectors along x-axis, y-axis and z-axis respectively. What does the statement express symbolically? 5+1=6

GROUP—B

(Mechanics)

(*Marks* : 40)

4. (*a*) One cannot experience fictitious force in centre of mass frame. Why? 1

(*b*) Show that gravitational force is a conservative force. 1

(*c*) Like velocity, momentum of a particle is also a reference frame dependent quantity. State the condition for which the momentum of the system can be made always zero for any inertial frame of reference. 1

(*d*) What are the two ways that a conservative force can be defined? 1

(*e*) State work-energy theorem. 1

(*f*) Write the differential equation for the motion of the pendulum. 1

5. (*a*) Determine the moment of inertia of a CO_2 molecule about an axis passing through its centre of mass. Given $m_e = 12$ a.m.u., $m_0 = 15 \cdot 985$ a.m.u., bond length $= 1 \cdot 15$ Å. 2

(b) The position of a moving particle at an instant is given by $\vec{r} = a\cos\theta\,\hat{i} + a\sin\theta\,\hat{j}$. Show that the force acting on the particle is conservative. 2

6. Answer any *two* questions : 5×2=10

(a) Find the gravitational potential produced by the two equal masses separated by a given distance. Show graphically how this will vary.

(b) Calculate the centre of mass of a solid hemisphere.

(c) Find out the mathematical expression of moment of inertia of the earth about its axis of rotation considering necessary assumption.

7. Answer any *two* questions : 10×2=20

(a) Give the theory of compound pendulum. Show the reversibility of the centre of suspension and the centre of oscillation of a compound pendulum. Find out the expression for minimum time-period of the pendulum. 4+3+3=10

(b) Explain the nature of the frame of reference attached to the falling body under gravity as observed by an observer standing on earth. Derive an expression for all the fictitious forces acting on a particle moving with a velocity in a rotating frame which is itself rotating with a uniform angular velocity with respect to another inertial frame. 2+8=10

(c) (i) Show that the angular momentum of an extended system is

$$\vec{L} = \vec{L}_{cm} + \vec{R}_{cm} \times \vec{M}_{cm}$$

where the symbols used are carrying their usual meanings.

(ii) A rigid body of solid symmetry is allowed to roll down an inclined plane without slipping. Show that the linear acceleration of the body is

$$\frac{g \sin\lambda}{1 + k^2/a^2}$$

where g is the acceleration due to gravity, k is the radius of gyration and a is the radius of the body rolling down the inclined plane making an angle λ with the horizontal. 4+6=10

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